Financial Trading Systems, Case Re1: Example prep.:

Suppose Traders A and B are trading the same asset (delivering the same dividend payoffs across possible states), but with different information sets:

At the outset of period 1, the traders have different (period 1, period 2) information[1]:

Trader A's information set: (not x, not x)

Period1\Period 2	X	Y	Z
X	~~0,0~~	~~0,12~~	~~0,24~~
Y	~~12,0~~	12,12	12,24
Z	~~24,0~~	24,12	24,24

Min. payoff:	$24
Max. payoff:	$48
Expected value:	$36

Trader B's information set: (not z, not z)

Period1\Period 2	X	Y	Z
X	0,0	0,12	~~0,24~~
Y	12,0	12,12	~~12,24~~
Z	~~24,0~~	~~24,12~~	~~24,24~~

Min. payoff:	$0
Max. payoff:	$24
Expected value:	$12

If we had the information sets of Traders A and B, we would of course have complete information, and rationally value the security according to the (12,12) payout across periods 1 and 2—i.e, without a discount rate, the security would be worth: $12+$12=$24. But neither trader has perfect information (at least at first), so let's consider the perspective of Trader B.

Suppose Trader B submits an Ask at $24, the asset's highest possible value according to her information set, and observes that Trader A is willing to buy ($24 is the lowest possible value according to Trader A's information). Trader B deduces that Trader A, if rational, cannot possibly share the same information set. Specifically Trader B deduces, Trader A's information must be:

[1] Example assumes a zero discount rate, and uses payoff matrices that differ from the Case's actual matrices.

Possible complementary information sets:

(not x, not y):	Min.: \$12; Max.: 48; Exp: \$30
(not y, not x):	Min.: \$12; Max.: 48; Exp: \$30
(not x, not x):	Min.: \$24; Max.: 48; Exp: \$36
(not y, not y):	Min.: \$0; Max.: 48; Exp: \$24
(not z, not y):	Min.: \$0; Max.: 36; Exp: \$18
(not z, not x):	Min.: \$0; Max.: 36; Exp: \$24
(not x, not z):	Min.: \$12; Max.: 36; Exp: \$24
(not y, not z):	Min.: \$0; Max.: 36; Exp: \$18

First, we can rule out (not z, not y) and (not y, not z), since Trader A is willing to pay \$24, which would exceed the expected payoffs in those sets. This leaves:

(not x, not y):	Min.: \$12; Max.: 48; Exp: \$30
(not y, not x):	Min.: \$12; Max.: 48; Exp: \$30
(not x, not x):	Min.: \$24; Max.: 48; Exp: \$36
(not y, not y):	Min.: \$0; Max.: 48; Exp: \$24
(not z, not x):	Min.: \$0; Max.: 36; Exp: \$24
(not x, not z):	Min.: \$12; Max.: 36; Exp: \$24

Next, Trader B sees a few of the remaining information sets give expected payouts higher than \$24, and decides to raise her Ask to just below \$30. Through observing Trader A's continued willingness to buy at say \$29 (still below her expected value of \$36), Trader B further rules out Trader A having the bottom three information sets. This leaves:

(not x, not y):	Min.: \$12; Max.: 48; Exp: \$30
(not y, not x):	Min.: \$12; Max.: 48; Exp: \$30
(not x, not x):	Min.: \$24; Max.: 48; Exp: \$36

Last, Trader B raises her Ask further somewhere just below $36 and again observes Trader A's continued willingness to buy. Trader B thus determines Trader A must have information set (not x, not x).

By combining the two information sets (not x, not x) and (not z, not z), Trader B acquires the complete information (y, y) and so knows with certainty the asset's payouts will be ($12, $12) across the two periods. Trader B is thus always willing to buy(sell) at prices below(above) $24 during period 1, and always willing to buy(sell) at prices below(above) $12 during period 2. Notice as well, the market impact on overall price-discovery from Trader B having obtained perfect information—Trader B's profiting from her complete information strengthens the central tendency for price convergence to $24, the efficient market price.

Alternatively (just for practice) suppose Trader A had *not* been willing to pay more than $24, Trader B would still have determined other valuable information, since Trader A would then need to have one of the information sets: (not y, not y), (not z, not x) or (not x, not z). Consider the pricing implication of these sets in combination with Trader B's information (not z, not z). Trader B would rationally determine the following: 1) In period 1, the asset's true total value must then be either, $0, $12, or $24. So selling the asset at it's true maximum value of $24 is great (but we already knew that); More importantly, observing an Ask of less than $12 (for example) from Trader A would rule out (not x, not z) as one of the remaining complementary information sets for Trader B, leaving only the other two sets as possibilities. From this Trader B would know that the asset's total value is either $0 or $12; 2) observing the actual period 1 payout always sharpens the information set when there is uncertainty. Consider the pricing implications of observing a $0 or $12 period 1 payout under this scenario for example.