



FTS Real Time System Project: Arbitrage Free Pricing and Carry Costs: Application of the Cost of Carry Model to Stock Index Futures

Question: How do you apply the cost of carry model of arbitrage free pricing to stock index futures?

Remark: In the first two future's assignments you learned how they are traded and then applied this to the real world problem of managing general market risk. In this assignment the objective is to understand how futures are priced. In particular, you are introduced to the cost of carry model for arbitrage free futures' pricing.

Stock Index Futures Small Group Project: Objectives

The objective of this project is to apply the cost of carry model for arbitrage free futures pricing of stock index futures defined on the S&P 500 index and the Nasdaq 100 index. The cost of carry model identifies the arbitrage free price of derivative contracts. It results from constructing the synthetic or replicating position of the derivative contract, in the cash markets and maintaining this replication position over the life of the derivative contract. It is this latter maintenance problem that generates the "carry costs" associated with the synthetic or replicating position.

In this project your task is to apply this model by addressing the following set of specific questions that arise in relation to the general motivating question. These are:

- (1) What is the cost-of-carry model and how do you estimate its inputs?

- (2) Are the prices obtained through cost-of-carry model the same as the real market prices?
- (3) Are there any exploitable arbitrage opportunities between the theoretical cost-of-carry model prices and the real market prices?

By completing this project you will see how the model works under perfect and imperfect market assumptions, and how the market conditions affect the security price and arbitrage opportunities.

1. Cost-of-Carry model

Mathematically this model is presented in one of three forms that differ in terms of the unit of measure for its inputs:

$F = S(1+(r+c-d)T)$	for simple interest rates (referred to as money market rates)
$F = S e^{(r+c-d)T}$	for continuous compounding rates
$F = S(1+r+c-d)^T$	for discrete time compounded rates
$F = S(1+(r+c-d)N/360)$	for “add-on” interest, also called the LIBOR basis

Here, F is the future price, S is the spot index price, r is the risk-free rate (financial carry cost), c is the cost of carry arising from physical storage, d is the dividend yield and T is the time to maturity and in the last equation, N is the number of days to maturity. Note that the first and the last are essentially the same, except that the LIBOR basis uses 360 days in a year. In each of the above equivalent forms, the left hand side is the derivative contract and the right hand side identifies the synthetic or replicating position using the spot markets.

Interpretation of the Models

Observe that the above model merely predict that the futures price equals the “future value” of the underlying index using a discount rate equal to the implied carry cost rate (net of the interest rate, dividend yield and storage costs express as a rate relative to the underlying index). For stock index futures there are no carry costs. As a result, the two important carry costs are the financial cost of carry r , and dividend yield d .

2. Important Operational details: Market Imperfections

Financial Cost of Carry and Interest Rate Conventions

The three forms of the model described in the previous section differ in terms of the interest rate convention applied. These are, simple interest, discrete compounded interest and continuous compounded interest.

For simple interest rates the interest is usually provided in terms of “Annual Percentage Rate” (APR). Similarly, compounded rates, either discrete or continuous, are also provided in APR form. The terminology used is often “Nominal versus Effective APR.”

Nominal APR = Simple Interest Rates

Effective APR = Compounded Interest Rate

Examples:

Total Nominal Interest = Simple Interest Rate (per annum) * Principal * Number of years

Total Effective Interest = Principal * (1 + Effective Interest Rate (per annum))^(Number of years)
- Principal

Compounding Note: Interest rates are usually expressed in APR form. However, the compounding frequency for compounded rates will vary. This is because the compounded period is often shortened from 1-year to multiple times a year. Lets denote this as “m” for multiple times but leave the rate still expressed in terms of APR.

Example: Suppose compounding is 4 times a year, every three months. That is $m = 4$. Alternatively, compounding may be monthly. There are twelve months in a year so $m = 12$. In general the total effective interest when the compounded frequency changes is as follows:

Total Effective Interest = Principal*(1 + Effective Interest Rate (APR)/m)^(number of years*m) – Principal

Finally, if the compounding frequency is continual we describe this as continuous because “m” approaches infinity. The limiting case of discrete compounding is continuous compounding and the Total Effective Interest equals:

Total Effective Interest (Continuously Compounded) = Principle*Exp(Numbers of years*Continuous Compounded Rate (APR))

Examples:

Suppose the interest rate is 10% APR simple interest, 10% compounded annually, 10% compounded monthly and 10% compounded continuously. Common sense would imply that the more frequently the same rate of interest is compounded the larger the investment will grow to. Consider what \$100 grows to at the end of the year:

Simple Interest: $100 * 1.10 = \$110$.

Compounded Annually = $100 * (1.10^1) = \$110$ (the same for 1 year only)

Compounded Monthly = $100 * (1 + .10/12)^{(12 * 1)} = \110.4713

Compounded Continuously = $100 * \text{Exp}(0.10 * 1) = \110.5171

That is, continuous compounded rate gives \$0.5171 more than simple.

Finally, even APR requires some attention. You are used to thinking of 1 year equaling 365 days in a non leap year. But in finance a year can be defined differently. The money market conventions for the US define one year as 360 days. For example, the LIBOR (London Bank Offered Rate) for USD is not only provided in *simple interest form* but it is quoted relative to a 360 day year. This is not a convenient form for valuation purposes and so you usually need to

convert a money market rate such as LIBOR to a discrete or continuous compounding rate in the form of a 365 day APR.

Example:

Suppose LIBOR = 5% and the day count is 360 then at the end of 365 days \$1 grows to $(1 + 0.05 \cdot (365/360)) = \1.050694

The implied continuous compounded rate (annualized) = 0.0494513 APR = $\ln(1.050694)$ which is less than the simple LIBOR rate of 5% relative to a 360 day count.

Libor is important for futures pricing because this is the rate that commercial banks can borrow and lend at and so this rate arguably provides a better description of the financial cost of carry in the real world as opposed to the “textbook world.” In later FTS assignments you can examine this particular issue directly. In this current assignment your objective is to become acquainted with how this model is applied in practice.

To estimate the financial cost of carry look up the current Libor rate for a maturity that approximates the remaining life of the futures contract you are working with. The FTS Real Time Client will provide you with Libor support.

Financial Cost of Carry Summary: In the FTS Real Time case the compounding convention is provided in the online support and this will be annual compounded rates. That is, in the US money markets, such as LIBOR, are quoted in simple interest form relative to a day-count convention (using 360 days as a year). For valuation purposes we usually work with either discrete or continuous compounding forms. As a result, your first task is to look up current Libor rates with a maturity that approximates the remaining life of the futures contract. You will use the FTS Real Time Client’s support system to access Libor rates and then you should convert these rates from the simple quoted rate into an equivalent “annual compounded rate.”

Example Financial Cost of Carry

Suppose the financial cost of carry for December 2010 contract is well approximated from the average of the 1-month and 3-month Libor rates. By clicking on the December Futures ticker then the following support appears in the top RHS of the screen:

Under Libor you have access to current Libor quotes. For example by clicking on the sub title Libor brings up the following type of screen:

The screenshot shows a trading software interface with a menu on the left and a data table on the right. The menu includes 'Charts', 'Dividends', and 'LIBOR', with 'LIBOR' expanded to show 'Bloomberg', 'FedReserve', and 'LIBOR'. The data table is titled 'Current LIBOR Rates' and has two columns: 'Maturity' and 'Rate (%)'. The table contains two rows of data: 'One-Month LIBOR' with a rate of 0.25625, and 'Three-Month LIBOR' with a rate of 0.28906.

Current LIBOR Rates ⁵	
Maturity	Rate (%)
One-Month LIBOR	0.25625
Three-Month LIBOR	0.28906

You can compute the average in Excel to be: $0.272655 (= 0.25625+0.28906)/2$.

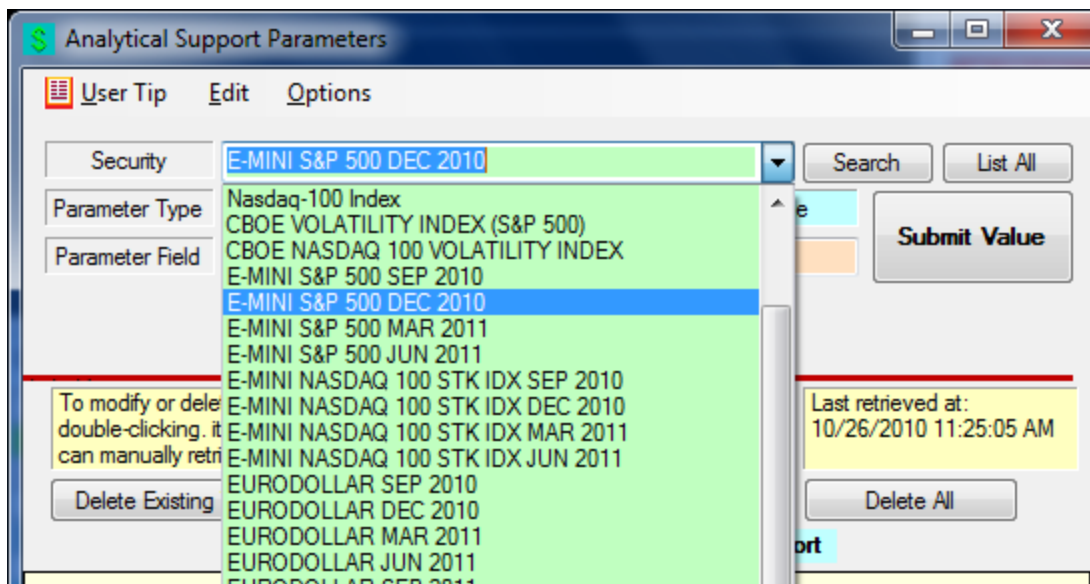
Suppose now you want to annualize this estimated 2-month rate (assume libor quotation is for 60-days relative to a 360 day year):

$$1 + \text{Annualized Rate} = (1 + 0.00272655 * (60/360))^{(365/60)} = 1.002768$$

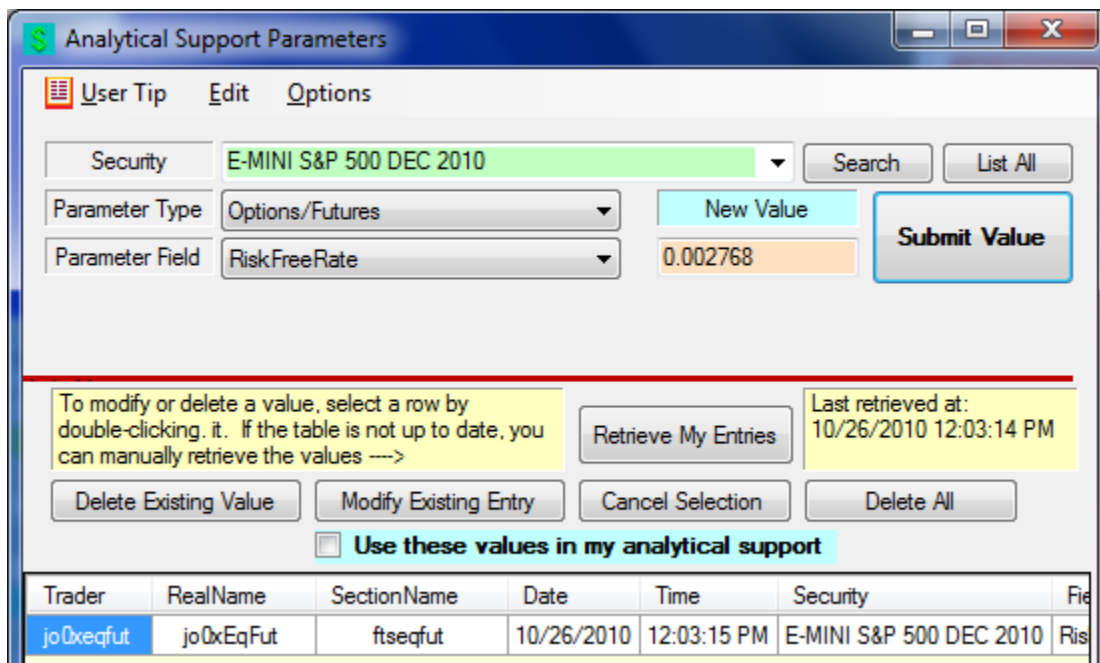
Thus the financial cost of carry estimated for the December contract is 0.2768%.

You can enter this into your personalized Parameter support as follows:

1. Click on the Parameter menu item at the top of the RTFTS screen.
2. Click on List All to list all of the securities and then select the December Stock Index Futures contract from the drop down as illustrated:

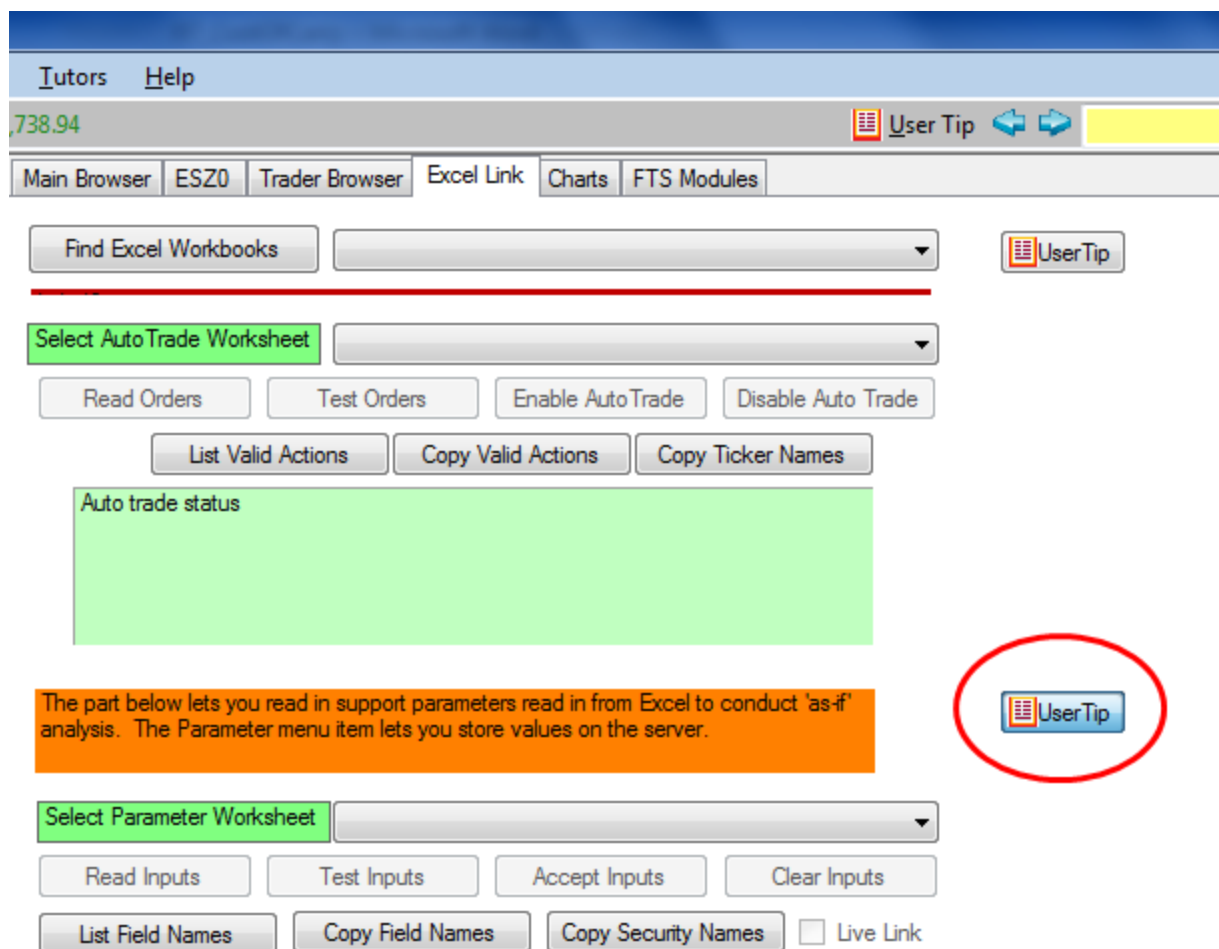


Now you are ready to personalize your parameter support:



Enter the number into the text box as above and click on Submit Value. Now you will see your value listed under Trader details as above.

Tip: You can also enter many at once using Excel see the top RHS of the screen and click on User Tips:



The user tips will provide online help with respect to how.

Finally in the Parameters window click on Use these Values in My Analytical Support and the analytical support now responds with your values:

The screenshot shows the 'Analytical Support Parameters' window. The 'Security' dropdown is set to 'E-MINI S&P 500 DEC 2010'. The 'Parameter Type' dropdown is set to 'Options/Futures'. The 'Parameter Field' dropdown is set to 'RiskFreeRate'. The 'New Value' input field contains '0.002768'. There are buttons for 'Search', 'List All', and 'Submit Value'. A yellow informational box contains text about modifying or deleting values and a 'Retrieve My Entries' button. Below this is a row of buttons: 'Delete Existing Value', 'Modify Existing Entry', 'Cancel Selection', and 'Delete All'. A checkbox labeled 'Use these values in my analytical support' is checked. At the bottom, a table shows a list of entries with columns: Trader, RealName, SectionName, Date, Time, Security, and Field. The first row is highlighted in blue.

Trader	RealName	SectionName	Date	Time	Security	Field
jo0xeqfut	jo0xEqFut	ftseqfut	10/26/2010	12:03:15 PM	E-MINI S&P 500 DEC 2010	Ris

To gain a stronger understanding of the subject matter you are encouraged to enter your own personal support parameters.

Dividend Yields

The second important input is the dividend yield for the underlying index. Again, this is not straightforward. The model only cares about dividends that will be paid over the life of the futures contract which is usually much less than 1-year. Thus the annualized dividend yield for the underlying index is only a first pass estimate. You need to further refine this estimate to *forecast* the dividends that are expected to be paid over the life of the futures contract. Fortunately, again this information is available in the FTS Real Time Client's support system which provides links to dividend projections for the life of the futures contracts. Your exercise will be to work with these projections, or your own, and then convert them into a form required by the cost of carry model. Later in this project you will compare your estimate with what you infer from the market's estimate.

Underlying Index

The third important input is the underlying index value. Again in the real world this number changes in real time during the trading hours and is freely available for the S&P500 and Nasdaq 100 indexes.

RTFTS Comprehensive support System

The FTS Real Time Client provides a comprehensive set of support links that include the following:

- i. Extensive dividend information relating to the futures contracts trading on the S&P500 and Nasdaq 100 indexes.
- ii. Similarly, for the case of the financial cost of carry again in a world with market imperfections, important decisions have to be made. If the world is best described by “perfect markets,” the best source of information about the financial cost of carry is provided from the US Treasury yield curve. The FTS Real Time Client provides you with immediate access to the US Treasury yield curve
- iii. In the presence of market imperfections the precise contender for the financial cost of carry again becomes blurred. A guiding rule is to focus on the financial cost of carry for the institution that is most likely to engage in arbitrage transactions. This is unlikely to be the US Treasury, but rather the major investment banks. The credit rating of these institutions is lower than the US Treasury, historically around “AA” (or worse). These institutions are able to borrow or lend at either the London Interbank Offered Rate or the US Treasury REPO rates. As a result, a stronger case can be made that these rates provide a better measure of the financial cost of carry. In the FTS Real Time trading case supporting this assignment extensive access is provided to the financial cost of carry including the Eurodollar futures markets.
- iv. Finally, to implement the arbitrage required to support the cost of carry model requires being able to form a replicating or synthetic position. In particular, this requires buying or selling the underlying index in the cash or spot markets. The underlying indexes are both broad based and their implied weights change in real time. Further, synchronous trading of all securities making

up the index is not possibly at a given instant of time. That is, again market imperfections imply that this is not precisely possible. In this project you have access to buying or selling a proxy for the underlying index, the Exchange Traded Funds (SPY and QQQQ). As a result, an additional source of risk is introduced into the exercise which is basis risk. Basis risk arises when the underlying is not tradable but instead what is tradable is highly correlated to the underlying but not perfectly correlated. Again the FTS Real Time Client provides you with comprehensive access to these markets.

Project Requirements:

All projects should contain a cover sheet that list the full name of each team member.

Required questions:

- (1) In your own words what is the cost-of-carry model and in practice how did your team estimate the inputs to this model?
- (2) Given your answer to (1) are the prices obtained through your application of the cost-of-carry model the same as the real market prices?
- (3) Given your answers to (1) and (2) were there any exploitable arbitrage opportunities between your assessed arbitrage free prices and the real market prices?

Phase I:

There are three parts to your phase I report. Part A – An Executive Summary (this is a summary not exceeding 1-page of your answers to the above three questions. Part B is the main body which should not exceed 5-pages to answer the above questions. Part C (optional) and additional information that you want to provide in appendices.

Your report should be logically organized and laid out clearly as it will be graded on both the *form and content* of the presentation.

The FTS Real Time trading case “Stock Index Futures” contains the details of the available markets, relevant web links and a comprehensive interactive analytical support system for working with the cost of carry model in the real world.

