



FTS Real Time Client: Performance Measures

Question: How is portfolio performance measured?

Introduction

Portfolio performance measures try to answer the question: how well did you do? The measures are used in several ways. For example, they are used to compare money managers: is one manager “better” than another? They are also frequently compared to “the market” which is how people back up claims that “they consistently outperform the market” or make arguments to the effect that “you cannot consistently beat the market.” To understand these statements, we need to know what it means to outperform or beat the market, and also what exactly is “the market.”

One way to measure performance is by the return. If I make 9% over one year and you make 10%, then you outperform me. If “the market” returns 7%, then both of us beat the market. A drawback of simply using returns is that it does not account for risk. If I undertook a very risky strategy and made 9% while you had a safer strategy that made 10%, then you outperformed me on two dimensions, risk and return. Even if you had only made 7% or 8% with a safer strategy, we may still feel that you outperformed me on a risk adjusted basis, i.e. that you had higher returns per unit of risk you took. A performance measure that accounts for risk is called a risk-adjusted performance measure.

The FTS Real Time Client has several performance measures built in. These are calculated both using expected or forward-looking information as well as ex-post, from historical performance. This note explains these measures.

The Market

A broad based market index, such as the S&P 500 index, is typically used as a measure of how the “market” performed. When you are measuring performance of a certain type of investment strategy, e.g. one that invests in energy stocks, then it may be more relevant to use an index of energy stocks. There are many indexes, by sector, industry, geography, and so on that can be used to determine the performance of “the market.” In the notation below, the subscript m refers to the market index.

The Measures in Theory

Sharpe Ratio: The most widely used risk-adjusted performance measure is the Sharpe Ratio. This ratio is defined by:

$$S = \frac{r - r_f}{\sigma}$$

Here, r is the return, r_f is the risk free return, and σ is the volatility (or standard deviation) of the return. It measures the amount by which you beat the risk free return per unit of risk.

Treynor Ratio: The Treynor Ratio is defined as

$$T = \frac{r - r_f}{\beta}$$

So here, risk is measured by the portfolio beta.

Jensen's Alpha:

$$A = r - (r_f + \beta * (r_m - r_f))$$

Here, r_m is the return on the market index. Jensen's alpha measures the amount by which you outperform the return predicted by the Capital Asset Pricing Model (CAPM).

M-Squared: The M2 performance measure is

$$M2 = r_f + (r - r_f) * \frac{\sigma_m}{\sigma}$$

Note that $M2 = r_f + S * \sigma_m$ where S is the Sharpe ratio. It re-scales the Sharpe ratio to the volatility of the market index. Think of it this way: suppose you increase or decrease your investment in the risk free asset so that the volatility of your portfolio equals that of the market. M2 is the return on that portfolio, so a higher M2 means higher risk adjusted performance.

Value at Risk

Finally, a more recent measure of risk is VaR, which stands for Value at Risk. The 95%-VaR is the value so that with 95% probability, you will not lose more than that amount. The FTS Real Time Client reports both the 95% and 99% VaR.

In Practice

In practice, the measures are used in two ways: *ex-ante* and *ex-post*. Ex-ante means "beforehand" and ex-post means "after the fact." Consider the Sharpe ratio. If we invest money for the next year, what matter are the expected return and the volatility of the return over the next year, i.e. looking forward. So r would be replaced by the *expected* return from the investment and σ would be the volatility of this expected return, again looking forward.

At the end of the year, we can calculate what actually happened. In this case, we would use the average realized return for r and the realized volatility of these returns for σ .

In the FTS RT Client, the analytical support screen for stocks shows you the ex-ante calculations. These require an estimate of expected return and so on for each stock:

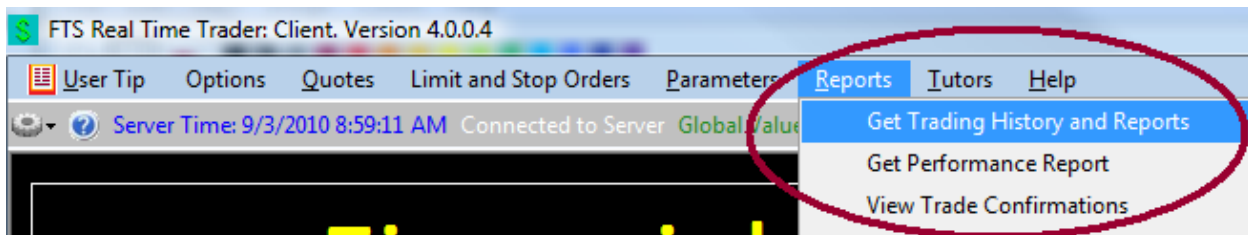
		Value	Beta	Exp Return	Volatility	Sharpe	Treynor	Jensen	M2	95% VaR
Overall Position		102,234.00	0.9830	0.0814	0.2806	0.1584	0.0452	0.0000	0.0821	2,994.03
Weight* (SS)			1.5100	0.1052	0.4308	0.1585	0.0452	0.0000	0.0821	4,595.94
Weight* (no SS)			1.5100	0.1052	0.4308	0.1585	0.0452	0.0000	0.0821	4,595.94
Eq Weighted			1.0183	0.0830	0.2912	0.1581	0.0452	0.0000	0.0820	3,107.04
Benchmark			1.0000	0.0821	0.2853	0.1585	0.0452	0.0000	0.0821	3,043.7644
Position (Raw)			0.9830	0.0814	0.2806	0.1584	0.0452	0.0000	0.0821	2,994.03
Name	Benchmark	Value	Beta	Risk Free Rate	Equity Premium	Exp Return	Volatility	Weight	Raw Weight	Weight* (SS)
ALCOA	S&P 500 INDEX	1,208.00	2.1100	0.0369	0.0452	0.1323	0.3608	0.0118	0.0118	0.0919
AMERICAN EXPRESS	S&P 500 INDEX	8,602.00	2.1700	0.0369	0.0452	0.1350	0.3469	0.0841	0.0841	0.0949
AT&T	S&P 500 INDEX	0.00	0.6600	0.0369	0.0452	0.0668	0.1436	0.0000	0.0000	0.0287
BANK OF AMERICA	S&P 500 INDEX	2,622.00	2.4500	0.0369	0.0452	0.1477	0.4131	0.0256	0.0256	0.1069
BOEING	S&P 500 INDEX	0.00	1.3100	0.0369	0.0452	0.0961	0.3070	0.0000	0.0000	0.0573
CATERPILLAR	S&P 500 INDEX	7,999.00	1.8200	0.0369	0.0452	0.1192	0.2918	0.0782	0.0782	0.0794
CHEVRON	S&P 500 INDEX	16,256.00	0.6900	0.0369	0.0452	0.0681	0.1944	0.1590	0.1590	0.0301

The analytics come in two varieties. You can use CAPM to calculate the expected return or you can specify your own:

		Value	Beta	Exp Return	Volatility	Sharpe	Treynor	Jensen	M2	95% VaR
Overall Position		102,234.00	0.9830	0.0890	0.2806	0.1858	0.0530	0.0077	0.0899	2,994.03
Weight* (SS)			0.0108	0.0394	0.0032	0.7906	0.2337	0.0020	0.2625	34.15
Weight* (no SS)			0.5910	0.0690	0.1690	0.1901	0.0544	0.0054	0.0912	1,802.48
Eq Weighted			1.0183	0.0908	0.2912	0.1852	0.0530	0.0079	0.0897	3,107.04
Benchmark			1.0000	0.0821	0.2853	0.1585	0.0452	0.0000	0.0821	3,043.7644
Position (Raw)			0.9830	0.0890	0.2806	0.1858	0.0530	0.0077	0.0899	2,994.03
Name	Benchmark	Value	Beta	Exp Return	Volatility	Weight	Raw Weight	Weight* (SS)	Weight* (no SS)	
ALCOA	S&P 500 INDEX	1,208.00	2.1100	0.1465	0.3608	0.0118	0.0118	-0.1345	0.0000	
AMERICAN EXPRESS	S&P 500 INDEX	8,602.00	2.1700	0.1496	0.3469	0.0841	0.0841	-0.1411	0.0000	
AT&T	S&P 500 INDEX	0.00	0.6600	0.0726	0.1436	0.0000	0.0000	0.1953	0.0464	
BANK OF AMERICA	S&P 500 INDEX	2,622.00	2.4500	0.1639	0.4131	0.0256	0.0256	-0.1998	0.0000	
BOEING	S&P 500 INDEX	0.00	1.3100	0.1057	0.3070	0.0000	0.0000	0.0428	0.0000	
CATERPILLAR	S&P 500 INDEX	7,999.00	1.8200	0.1317	0.2918	0.0782	0.0782	-0.0719	0.0000	
CHEVRON	S&P 500 INDEX	16,256.00	0.6900	0.0741	0.1944	0.1590	0.1590	0.1836	0.0000	
CISCO SYSTEMS	S&P 500 INDEX	0.00	1.2400	0.1021	0.4451	0.0000	0.0000	0.0001	0.0000	

The Parameters window lets you change the estimates of expected return, beta, the equity premium, volatility, and so on. These are all ex-ante.

Ex-post, you have to look at what actually happened by obtaining your history:



You can then generate your performance report:

The screenshot shows a window titled "Trading History" with a menu bar containing "User Tip", "Edit", "Options", and "Refresh History". Below the menu is a "Select Report" dropdown menu set to "Performance Report", with "Generate Report" and "Export Table to Excel" buttons. A green message box states: "The history is not refreshed in real time. If you make a trade or want to recalculate profits and losses, please Refresh History and then Generate Report." Below this, it says "Last retrieved at: 10/2/2012 7:36:38 AM". The main content is a table with the following data:

	Portfolio	Benchmark
Average Return	0.00000097	0.00066015
Volatility	0.00020341	0.00526031
Alpha	-0.00065918	
Tracking Error	0.00516150	
Risk Free Rate	0.00004514	
Sharpe Ratio	-0.21716411	0.11691492
Beta	0.01935520	1
Treynor	-0.00228222	0.00061501
Daily VaR (%)	0.00033562	0.00867951

These are calculated in raw units. Usually, your portfolio is marked to market daily, so these are all daily numbers. If you want to annualize them, you should multiply every return by 250 (since there are approximately 250 trading days in a year) and every volatility calculation by the square root of 250. The exact calculations are presented in the Appendix.

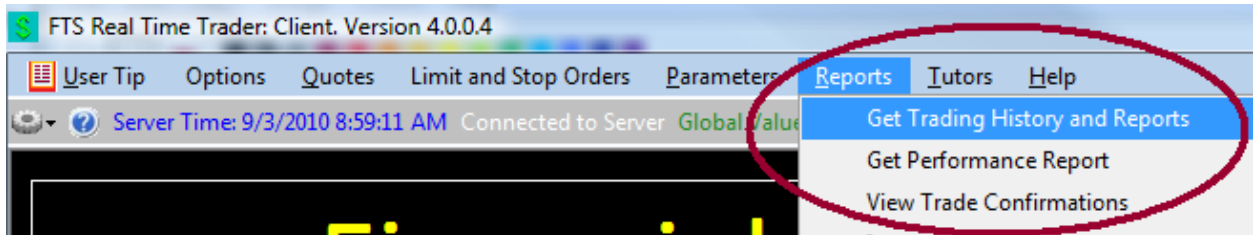
Exercises:

- Using any of the FTS Real Time Cases, after you have held a position for at least 10 days, verify the calculation of the Sharpe Ratio. Make sure you invest most of your cash, otherwise the cash portion of your portfolio will dominate the performance measure.
- If your portfolio beta was greater (less) than 1, was the portfolio volatility greater (less) than the volatility of the benchmark? Would you expect it to be?
- Calculate the M-Squared of your portfolio.
- Performance measures are frequently used to rank portfolio managers. For example, if the Sharpe ratio of Manager 1 is greater than the Sharpe ratio of Manager 2, then Manager 1 would be ranked higher than Manager 2 according to the Sharpe ratio.
 - Can Manager 1 have a higher Sharpe ratio but a lower M-Squared?
 - Can Manager 1 have a higher Sharpe ratio but a lower Treynor Ratio? Explain how this could occur and whether you expect that it could happen in practice.

5. Do you think risk adjusted measures like the Sharpe or Treynor Ratio's, provide a better measure of performance than simply the total return of a manager's portfolio?

Downloading the history

You download your trading history from the reports menu:



Once the history has been downloaded and processed, you would generate the report "End of Day Market Values:"

Trading History

User Tip Edit Options Refresh History

Select Report: End of Day Market Values Generate Report Export Table to Excel

Last retrieved at: 9/6/2010 7:39:50 AM

Table Graph

Date	Value	Return	Benchmark	Benchmark Return	Alpha
8/21/2010	1000000		1071.69		
8/22/2010	1,000,004.00	0.00000	1,071.69	0.00000	0.00000
8/23/2010	1,000,008.00	0.00000	1,067.36	-0.00404	0.00404
8/24/2010	1,000,012.00	0.00000	1,051.87	-0.01451	0.01452
8/25/2010	1,000,017.00	0.00000	1,055.33	0.00329	-0.00329
8/26/2010	999,705.30	-0.00031	1,047.22	-0.00768	0.00737
8/27/2010	1,000,806.00	0.00110	1,064.59	0.01659	-0.01549
8/28/2010	1,000,810.00	0.00000	1,064.59	0.00000	0.00000
8/29/2010	1,000,813.00	0.00000	1,064.59	0.00000	0.00000
8/30/2010	999,839.20	-0.00097	1,048.92	-0.01472	0.01375
8/31/2010	999,783.30	-0.00006	1,049.33	0.00039	-0.00045
9/1/2010	1,002,195.00	0.00241	1,080.29	0.02950	-0.02709
9/2/2010	1,002,097.00	-0.00010	1,090.10	0.00908	-0.00918
9/3/2010	1,003,633.00	0.00153	1,104.51	0.01322	-0.01169
9/4/2010	1,003,636.00	0.00000	1,104.51	0.00000	0.00000
9/5/2010	1,003,639.00	0.00000	1,104.51	0.00000	0.00000

Export these to Excel and calculate the average return and the standard deviation (or volatility) of your daily returns. Parameters needed for some of the calculations, such as beta or the risk free rate, may be copied from the analytics displays.

Appendix: How the ex-post measures are calculated

The ex-post performance measures are calculated from the daily market values of your portfolio. We now explain how each value is calculated. These calculations are also in the following Excel spreadsheet:

<http://www.ftsmo.du.edu/public/modules/ftsRT/textBook/ExPostPerformance.xlsx>

You can use that spreadsheet to check your calculations, and when reading this, it will be helpful to have the spreadsheet open. You can then see how the formulas below are implemented in Excel.

Note: since returns are calculated on a daily basis, the numbers can be very small, and so there will always be slight rounding error. You should take that into account when verifying the calculations.

Let V_t be the portfolio value at the end of day t , where $t=0,1,\dots,T$. This means we have the end of day market value for $T+1$ days. In the spreadsheet, the market value is reported for 65 days, so $T = 64$

Let r_t = portfolio return between day t and day $t-1$:

$$r_t = \frac{V_t - V_{t-1}}{V_{t-1}}$$

Note that we have a return calculated for T days; we use the first market value, V_0 , to calculate r_1 .

Then, the average return is

$$\bar{r} = \frac{1}{T} \sum_{t=1}^T r_t$$

The average return for the benchmark is calculated in the same way. Let B_t be the value of the benchmark on day t . Let rb_t be the return on the benchmark. Then,

$$rb_t = \frac{B_t - B_{t-1}}{B_{t-1}}$$

And

$$\bar{rb} = \frac{1}{T} \sum_{t=1}^T rb_t$$

The volatility is the standard deviation of the returns. It is calculated as:

$$\sigma = \frac{1}{(T-1)} \sum_{t=1}^T (r_t - \bar{r})^2$$

Note the division by T-1; this follows the practice in Excel's STDEV function for calculating a standard deviation.

Similarly, the volatility of the benchmark is

$$\sigma_b = \frac{1}{(T-1)} \sum_{t=1}^T (rb_t - \overline{rb})^2$$

The tracking error on day t is the amount by which the portfolio return beat the benchmark return, and is called the alpha (note that this calculation is different from Jensen's Alpha referred to earlier):

$$\alpha_t = r_t - rb_t$$

The tracking error is simply the standard deviation of the daily tracking error. If the tracking error is zero, then you have perfectly tracked the benchmark returns. Choosing a portfolio to minimize the tracking error is used, for example, by exchange traded funds that try to mimic and index such as the S&P500.

The Sharpe ratio is risk-adjusted rate excess return:

$$S = \frac{\bar{r} - r_f}{\sigma}$$

Here r_f is the risk free interest rate

Beta is defined as follows:

$$\beta = \frac{\text{covariance}(r, rb)}{\sigma^2}$$

The covariance is calculated as follows (COVAR function in Excel):

$$\text{covariance}(r, rb) = \frac{1}{(T-1)} \sum_{t=1}^T (r_t - \bar{r})(rb_t - \overline{rb})$$

The Treynor Ratio is

$$T = \frac{\bar{r} - r_f}{\beta}$$

The Daily VaR (%) is 1.65σ