FTS Real Time System Project: Application of the Option Pricing Model to Stock Index Options

Question: How do you apply the option pricing model to stock index options?

Stock Index Options Project: Objectives

The objective of this project is to apply the option pricing model for identifying the arbitrage free price of options defined on the S&P 500 index. The option pricing model identifies the arbitrage free price of option contracts. It results from constructing the synthetic or replicating position of the option contract, in the cash markets by constructing a position in the underlying asset and a riskless bond. By maintaining this replicating position over the life of the option contract, in theory a riskless hedge could maintained and which should earn the risk free rate. A major insight from the Black-Scholes Option Pricing Model was to demonstrate how. The "how" is referred to as delta hedging, and it implies that to attain the theoretical result the replicating position's weights must change continuously. In practice this is not possible but practice can approximate the theoretical results. In this sequence of projects you will gain important insights into how well you can approximate the theory n practice.

In this first project your task is to learn how to apply the Black Scholes Option Pricing Model (OPM) which immediately raises the following questions:

(1) What is the OPM and how do you estimate its inputs?
(2) Are the prices obtained through the OPM the same as the real market prices?

(3) Are there any exploitable arbitrage opportunities between the theoretical OPM prices and the real market prices?

By completing this project you will see how the model works under perfect and imperfect market assumptions, and how the market conditions affect the security price and arbitrage opportunities.

1. Option Pricing Model

A call option is a derivative contract that provides it's holder with the right to buy the underlying asset for the strike or exercise price over the life of the call option. Immediately this definition introduces many terms. First, consider the difference between a right and an obligation. If a legal obligation is created then you must satisfy this obligation irrespective of whether it makes you worse off or better off. This immediately raises a subtle point associated with options. There are two sides to the contract --- the buyer and the seller or option writer. Options are complicated from the fact that the legal requirements created depend upon which side you view the option from. When viewed from the option writer's perspective they are legally obligated to settle if the option buyer chooses to exercise their option. From the other side the option buyer has acquired the right to exercise the option at their discretion. For example, a buyer will exercise the option at the end of it's life if it is "in-the-money" so that the option writer gives the option buyer something that is worth more than what the buyer pays the writer. Clearly, this mis match in legal rights/obligations will not be free. As a result, the option price (referred to as the option's premium) is what the buyer pays the option writer for this right.

Mathematically the OPM presented below provides the theoretical price or premium for a European Call Option. A European Call Option is an option that can only be exercised at the end of its life. In the equation below C is the option price or premium that is predicted by the model.

$$C = SN(d_1) - Xe^{-rT}N(d_2)$$

$$d_1 = \frac{\log\left(\frac{S}{X}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

The equation looks complicated and its derivation is but it has a simple underlying structure. The left hand side is the predicted option price and the right hand side identifies the nature of the spot market replicating position for the option. That is, it consists of the asset and a riskless bond (face value equal to the strike price). The N(d's) provide the weights in the replicating position. If we ignore the N(d's) for a minute then the call option price equals the underlying spot price of the asset less the present value of the strike or exercise price. That is, theoretically this implies that you should not exercise a European call option early because you will pay the strike price X whereas the market will only reward you for the present value of X not X. This same insight holds even when we consider the two additional N(d) terms. These terms are probabilities that are applied to S and the present value of X because the underlying asset price is expected to change over the life of the option. As a result, the above model predicts that the arbitrage free price of the option depends upon five inputs:

S = Spot asset price

X = Strike or exercise price that must be paid by the buyer to the writer at time of exercise

σ = Volatility of the underlying asset

T = Remaining time to maturity for the option

r = The risk free rate

There is a sixth input in practice which is dividend yield (d) if the underlying pays a dividend. The dividend yield reduces the financial cost of carry r, if the underlying pays a dividend because an option is not dividend protected and holding the replicating position will now earn (r –d) not r as is the case for a non-dividend paying underlying asset.

The most surprising element of the above equation is that the underlying expected return from the underlying asset drops out! That is, there is little forecasting implied when applying the above model because the model only requires the spot price of the underlying not a forecast of all future stock returns. The main variable that needs to be forecast is the volatility of the underlying asset.

2. Important Operational details: Market Imperfections

The necessary inputs for the OPM are the risk-free rate, dividend yield and the underlying spot index value. When applying the OPM you will immediately find that market imperfections affect each of these inputs in a way that precludes you from immediately applying the above mathematical formula. By working with the FTS Real Time Client program you will learn from personal trading experience what problems immediately arise when applying the OPM to your trading position. The interactive analytical support will allow you to see the practical issues that arise from market imperfections in the real world. We first provide an overview of these issues.

From the above it is implied that trading options is a complex multidimensional problem. In particular, the option trader needs to understand how option prices are predicted to respond to changes in the underlying asset price, underlying asset price volatility, and time. The FTS Real Time Client provides a comprehensive set of support links that provide these important insights. This includes the following:

First select from the analytical support: Equity Options Portfolio Analytics

Edit | US Dollar | Equity Options: Portfolio Analytics (Implied) | Parameters | UserT

	Delta	Gamma	Vega	Theta																
Beta Weighted	0.0000	0.0000	0.0000	0.0000																
	Mkt Price	Underlying	Contracts	Imp Vol	User Vol	User Price	Implied Delta	Implied Gamma	Implied Vega	Implied Theta	Size	Intrinsic Val	Time Prem	Style	Type	Strike	Maturity	Risk Free	Div Yld
SPX Call Eur 1050 Mar 19 2011	131.50	1,165.15	0	0.2253	0.2071	127.2988	0.7569	0.0018	235.4026	-44.1383	100	115.15	16.35	Call		1,050.0000	3/19/2011	0.0033	0.0213
SPX Call Eur 1100 Mar 19 2011	104.00	1,165.15	0	0.2445	0.2071	93.4939	0.6448	0.0019	282.8473	-64.9983	100	65.15	38.85	Call		1,100.0000	3/19/2011	0.0033	0.0213
SPX Call Eur 1150 Mar 19 2011	73.20	1,165.15	0	0.2310	0.2071	65.9568	0.5389	0.0022	303.0490	-68.2760	100	15.15	58.05	Call		1,150.0000	3/19/2011	0.0033	0.0213
SPX Call Eur 1200 Mar 19 2011	47.10	1,165.15	0	0.2152	0.2071	44.6828	0.4203	0.0023	299.3787	-64.4896	100	0.00	47.10	Call		1,200.0000	3/19/2011	0.0033	0.0213
SPX Call Eur 1250 Mar 19 2011	26.00	1,165.15	0	0.1955	0.2071	29.0886	0.2921	0.0023	263.6221	-52.5678	100	0.00	26.00	Call		1,250.0000	3/19/2011	0.0033	0.0213
SPX Call Eur 1300 Mar 19 2011	12.50	1,165.15	0	0.1801	0.2071	18.2221	0.1758	0.0019	198.7304	-37.0845	100	0.00	12.50	Call		1,300.0000	3/19/2011	0.0033	0.0213
SPX Call Eur 900 Jun 18 2011	195.00	1,165.15	0	N/A	0.2071	256.3678	0.9220	0.0006	119.9258	1.8750	100	265.15	-70.15	Call		900.0000	6/18/2011	0.0036	0.0213
SPX Call Eur 950 Jun 18 2011	204.00	1,165.15	0	0.1501	0.2071	212.3964	0.9320	0.0007	104.7750	8.5206	100	215.15	-11.15	Call		950.0000	6/18/2011	0.0036	0.0213
SPX Call Eur 1000 Jun 18 2011	178.50	1,165.15	0	0.2320	0.2071	171.9942	0.7845	0.0012	269.7269	-28.6741	100	165.15	13.35	Call		1,000.0000	6/18/2011	0.0036	0.0213

We will work with part of the above screen first for expositional purposes:

Edit | US Dollar | Equity Options: Portfolio Analytics (Implied) | Parameters

	Delta	Gamma	Vega	Theta						
Beta Weighted	0.0000	0.0000	0.0000	0.0000						
	Mkt Price	Underlying	Contracts	Imp Vol	User Vol	User Price	Implied Delta	Implied Gamma	Implied Vega	Implied Theta
SPX Call Eur 1050 Mar 19 2011	131.50	1,165.15	0	0.2253	0.2071	127.2988	0.7569	0.0018	235.4026	-44.1383
SPX Call Eur 1100 Mar 19 2011	104.00	1,165.15	0	0.2445	0.2071	93.4939	0.6448	0.0019	282.8473	-64.9983
SPX Call Eur 1150 Mar 19 2011	73.20	1,165.15	0	0.2310	0.2071	65.9568	0.5389	0.0022	303.0490	-68.2760
SPX Call Eur 1200 Mar 19 2011	47.10	1,165.15	0	0.2152	0.2071	44.6828	0.4203	0.0023	299.3787	-64.4896
SPX Call Eur 1250 Mar 19 2011	26.00	1,165.15	0	0.1955	0.2071	29.0886	0.2921	0.0023	263.6221	-52.5678
SPX Call Eur 1300 Mar 19 2011	12.50	1,165.15	0	0.1801	0.2071	18.2221	0.1758	0.0019	198.7304	-37.0845
SPX Call Eur 900 Jun 18 2011	195.00	1,165.15	0	N/A	0.2071	256.3678	0.9220	0.0006	119.9258	1.8750
SPX Call Eur 950 Jun 18 2011	204.00	1,165.15	0	0.1501	0.2071	212.3964	0.9320	0.0007	104.7750	8.5206
SPX Call Eur 1000 Jun 18 2011	178.50	1,165.15	0	0.2320	0.2071	171.9942	0.7845	0.0012	269.7269	-28.6741

The first column above defines the option contract. For example, SPX Call Eur 1050 Mar 19 2011 provides the following information:

The option is SPX – this is the ticker used at CBOE (Chicago Board Options Exchange) for the S&P 500 index. Call refers to a call option and Eur refers to the option's exercise style being European. European implies that it can only be exercised at the end of its life. An alternative exercise style is American which can be exercised any day including the day of maturity. The strike price is 1050 and the time of maturity is March 19, 2011. The other columns are:

Market Price = Option Premium (Price) in the market $131.50

Underlying = Spot price of the underlying asset (e.g., current price of the S&P500 index = 1165.15)

Contracts = Number of contracts in your position (1 contract controls $100*S&P 500 Index or currently $100*1165.15 = $116515.00 of the underlying index)

Implied Volatility = the volatility implied from the OPM given market price, underlying S, strike price X, remaining time to maturity, risk free rate and dividend yield)

User Volatility = your estimate of the underlying volatility

User Price = predicted OPM price given your estimate for volatility. That is, the support automatically calculates the prediction option price given your personal inputs.

The next set of labels refer to the option "Greeks." These are model derived estimates and they represent the predicted sensitivity of the option price to important pricing variables. Just as with CAPM we refer to a stock's "beta" as a measure of sensitivity to the general market in the option pricing world we refer delta, gamma, vgega and theta as sensitivity measures for the option contract.

Implied Delta = Sensitivity to the underlying asset price. For a call option this is a number between 0 and 1 that is interpreted as the predicted price increase of the option given a $1 increase in the underlying asset price. This is an important number for constructing a replicating position for the option from a stock and a bond and a delta neutral position is a position that is predicted to be insensitive to the change in the underlying.

Implied Gamma = Sensitivity of Delta to a change in the underlying. In practice this provides a measure of how frequently a position that is designed around delta (e.g., a delta neutral position) needs to be adjusted.

Implied Vega = sensitivity to the asset's volatility. This is a positive number because volatility increases imply that the predicted price of an option (call or put) increases.

Implied Theta = sensitivity to time. This is a negative number because the call option is worth more the longer it's life. As a result, an option price decays with time.

The remaining fields are:

Size = Option contract – this is defined as $100*S&P 500 index

Intrinsic Value = Underlying asset price minus the strike price (no adjustment for discounting)

Time Premium = Difference between the Intrinsic Value and the option's premium

Style = European or American

Type = Call or Put

Strike = Strike price defined in the contract

Maturity = Time of maturity

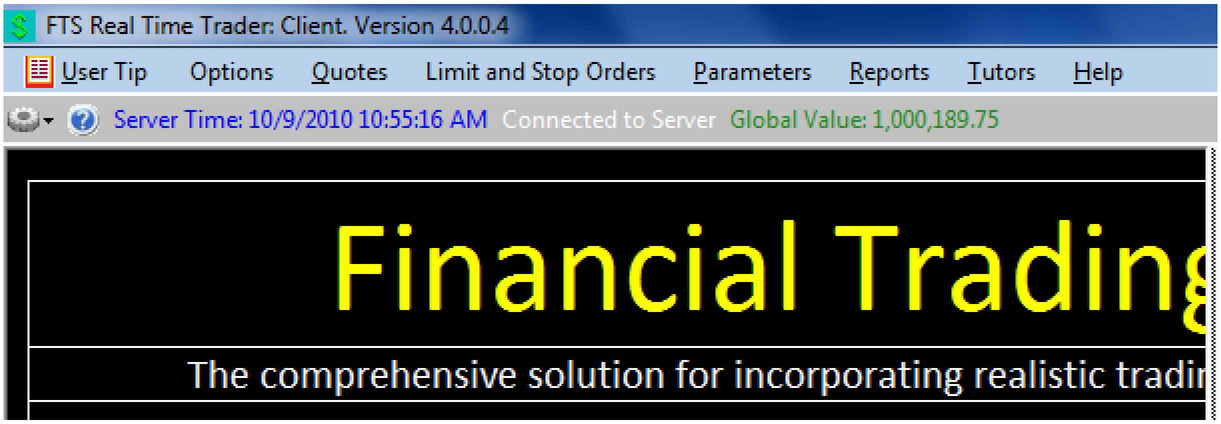
Risk Free = Financial cost of carry

Dividend Yield = Dividend yield for the underlying asset

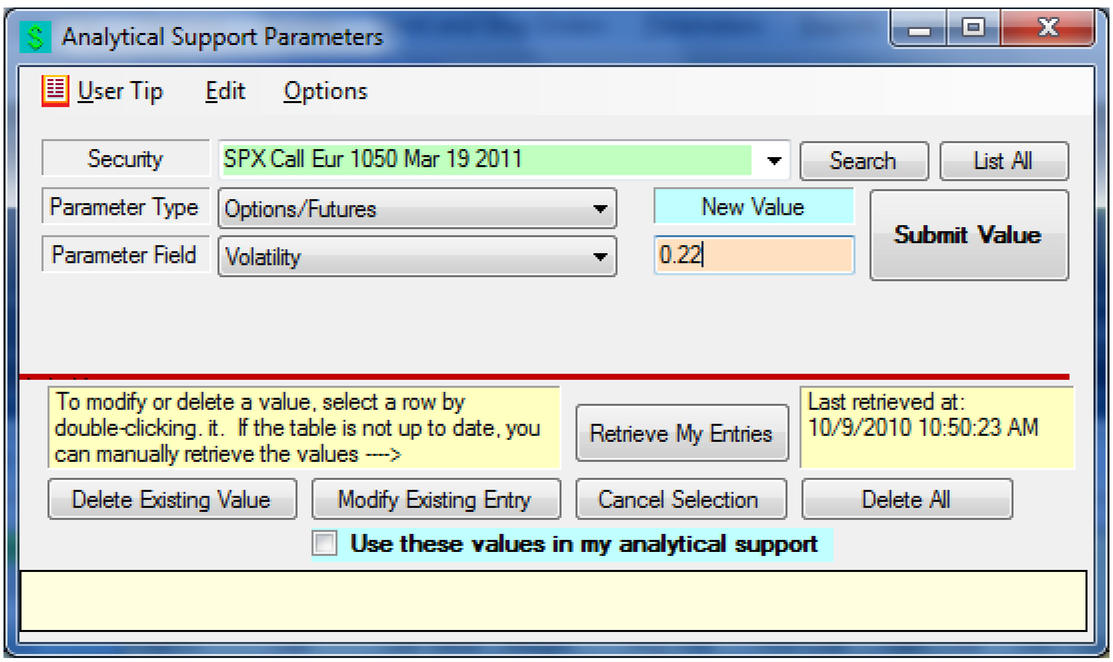
Working With the Interactive Support

The FTS Real Time Client's support system provides another additional powerful level of support. The fields described

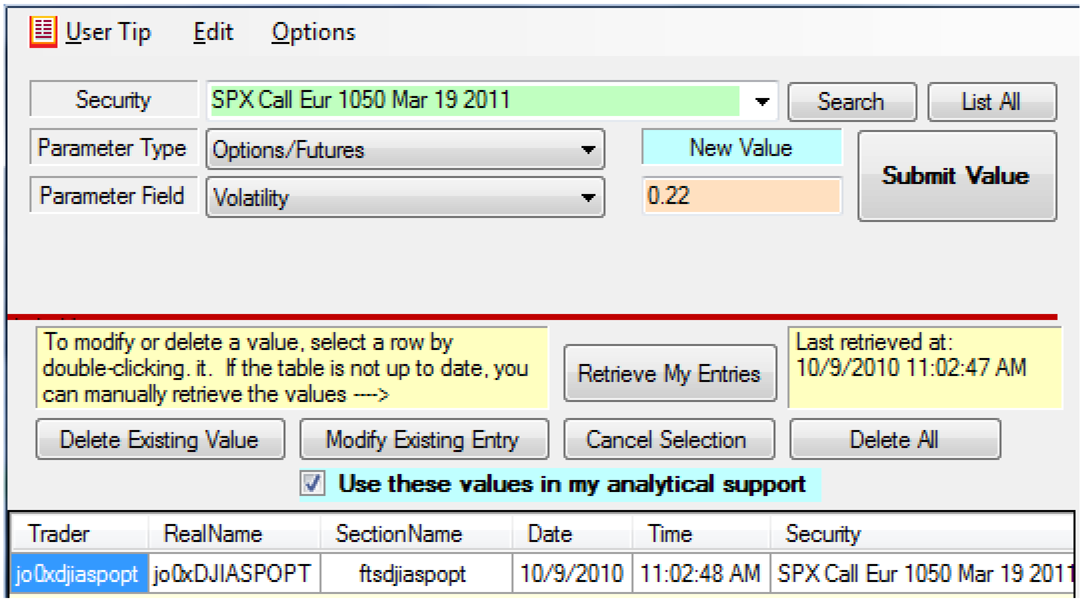
First select <u>P</u>arameters from the general menu item:



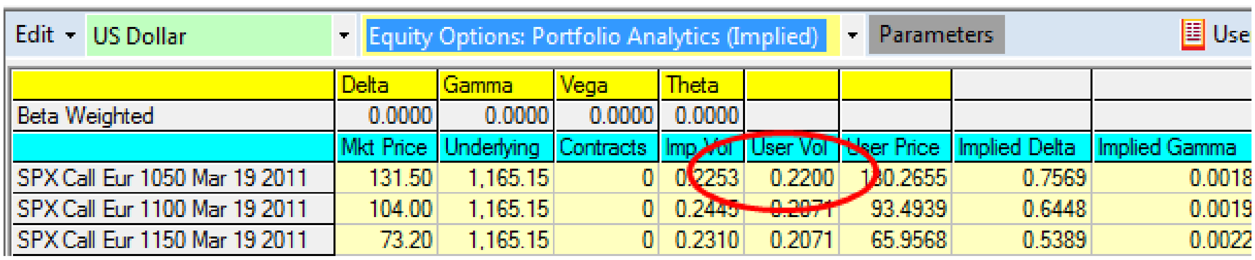
Second, select Parameters, List All and select the option contract:



For example, in the above suppose you want to change volatility to 0.22. In the above enter select the contract, enter volatility and click on submit Value. Observe there is a toggle switch beside the check box above: Use these values in my analytical support.



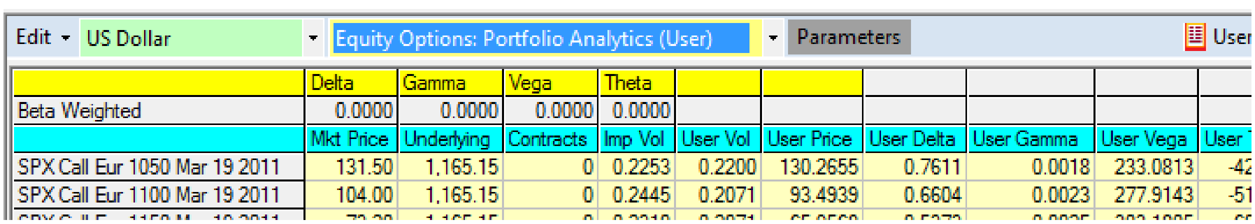
This controls Volatility in two of the option support systems:



	Delta	Gamma	Vega	Theta				
Beta Weighted	0.0000	0.0000	0.0000	0.0000				
	Mkt Price	Underlying	Contracts	Imp Vol	User Vol	User Price	Implied Delta	Implied Gamma
SPX Call Eur 1050 Mar 19 2011	131.50	1,165.15	0	0.2253	0.2200	130.2655	0.7569	0.0018
SPX Call Eur 1100 Mar 19 2011	104.00	1,165.15	0	0.2445	0.2071	93.4939	0.6448	0.0019
SPX Call Eur 1150 Mar 19 2011	73.20	1,165.15	0	0.2310	0.2071	65.9568	0.5389	0.0022

Alternatively you can select Equity Options: Portfolio Analytics (User):

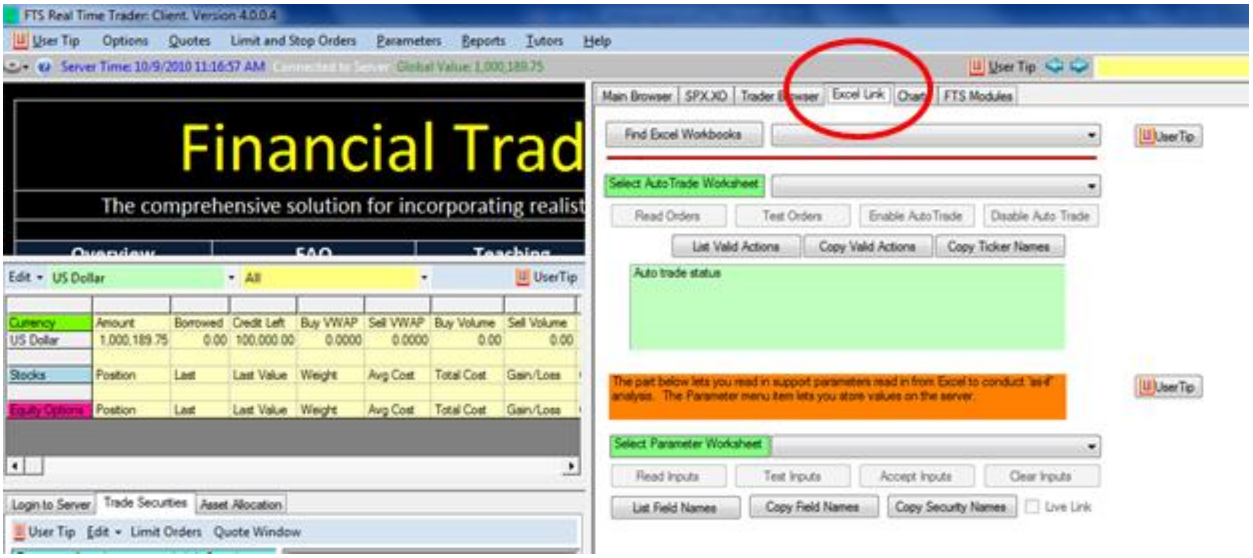
This gives more extensive user values – such as user implied greeks etc.,.



	Delta	Gamma	Vega	Theta						
Beta Weighted	0.0000	0.0000	0.0000	0.0000						
	Mkt Price	Underlying	Contracts	Imp Vol	User Vol	User Price	User Delta	User Gamma	User Vega	User T
SPX Call Eur 1050 Mar 19 2011	131.50	1,165.15	0	0.2253	0.2200	130.2655	0.7611	0.0018	233.0813	-42
SPX Call Eur 1100 Mar 19 2011	104.00	1,165.15	0	0.2445	0.2071	93.4939	0.6604	0.0023	277.9143	-51

For example, observe how User Delta relative to 0.22 for SPX Call Eur 1050 Mar is 0.7611 whereas in the implied support this is 0.7569. The difference between working with implied volatility from the current option price versus working with the user specified volatility of 0.22 in the current example.

Finally, changing parameters one at a time is very laborious so the FTS Real Time Client allows you to do this via Excel. First click on Excel Link in the screen below



By clicking on (bottom) User Tips in the above screen provides the information on how to change many Parameter inputs at once by linking to Excel.

Project Requirements:

All projects should contain a cover sheet that list the full name of each team member.

Required questions:

(1) In your own words describe the Option Pricing Model and in practice how did your team estimate the inputs to this model?

(2) Given your answer to (1) are the prices obtained through your application of the cost-of-carry model the same as the real market prices?

(3) Given your answers to (1) and (2) were there any exploitable arbitrage opportunities between your assessed arbitrage free prices and the real market prices?

Phase I:

There are three parts to your phase I report. Part A – An Executive Summary (this is a summary not exceeding 1-page of your answers to the above three questions. Part B is the main body which should not exceed 5-pages to answer the above questions. Part C (optional) and additional information that you want to provide in appendices.

Your report should be logically organized and laid out clearly as it will be graded on both the *form and content* of the presentation.

There are several FTS Real Time option trading cases so select the one your class has been assigned which contains the details of the available markets, relevant web links and a comprehensive interactive analytical support system for working with the OPM in the real world.