FTS Real Time System Project: Portfolio Diversification

Note: this project requires use of Excel's Solver

Question: How do you create a diversified stock portfolio?

Advice given by most financial advisors is to not put all your eggs in one basket, i.e. to diversify, meaning that you hold several different stocks. The reasoning behind this is simple; if you hold one stock or very few stocks, you can either do very well and very poorly. By diversifying, you can reduce the chances of extreme outcomes, and thus reduce the risk of your portfolio; of course, you typically also affect the return you expect to get.   How much you diversify typically depends on your risk aversion; the more you dislike risk, the more you would like to diversify.

Modern portfolio theory provides a technique for both measuring risk and return and determining the best way to diversify. In this project, you will learn to use Excel's Solver to create an "optimal" diversified portfolio. The details of modern portfolio theory are described in most investments textbooks, and will not be given here; instead, we will focus on the implementation, though we will need the following terminology.

Definitions

We start with a concept of a portfolio weight. It measures the proportion of your investment in a stock. For example, suppose you have two stocks in your portfolio, your position stock 1 is worth $1,000 and that in stock 2 is worth $3,000. Then, your total investment is worth $4,000, the proportion in stock 1 is 25% and the proportion in stock 2 is 75%.

Let w_i denote the portfolio weight of stock i. So in the example, $w_1 = 0.25$ and $w_2 = 0.75$.

Short Sales

If you only have long positions, then the concept of a portfolio weight is simple. It is not so simple when you short sell a stock. Let's modify the example. Suppose the price of stock 1 is 20 and you short sold 100 shares. Suppose the price of stock 2 is 40 and you hold 50 shares. What is the amount invested in stocks? If you simply add up the price times the position, you get zero: -20*100 + 40*50 = -2000 + 2000 = 0. What is the proportion in stock 1? It's not defined.   In fact, if we short sold 200 shares of stock 1,

the value calculated in this way would be -20*200 + 40*50 = -2000. If we simply divided, we would get a negative weight on stock 2 and a positive weight on stock 1.

The margin requirements for short sales complicate the picture a little bit more. When you short sell stock 1, the proceeds from the sale, $2000 in our example, are held in an account and you have to put up 50% of that as a deposit. So in this case, your position is:

Stock 1: -100 shares, each share worth $20, and $2000 + $1000 held in your account.

Stock 2: 50 shares, each worth $40

If you closed out your positions (so covered the short sale of stock 1 and sold stock 2), you would get $3000 back as follows: when you cover the short sale of stock 1, it costs $2,000 to buy back the shares. You had $3,000 in your account, so you get back $1,000. And you get $2,000 from the sale of stock 2.

Looking at it from the point of view, the dollar amount in your stock portfolio is $3,000, made up of $1,000 being the value of the position in stock 1 and $2,000 being the value of the position in stock 2. On this basis, the portfolio weights are 33% and 67%. These weights are sometimes called "adjusted weights."

Practical note: The dollar amount you would get if you closed out your position is sometimes called the "Net Liquidation Value" in brokerage accounts.

Note that when we take the short sale deposits etc. into account, the position value will always be positive and so the weight calculated on this basis will always be well defined. But the weight is unintuitive: we have short sold stock 1 but its weight is a positive number!

So we have a dilemma: if we report weights in a "natural" way, they may be undefined and unintuitive; if we define them taking after properly taking into account the short sale account, they are well defined but are unintuitive for short positions.

To resolve this, the FTS Real Time Client reports two weights: the *weight* is what takes into account the short sale (and margin) accounts properly; the value is calculated on the basis of what you would get back if you covered your position. The *raw weight* is calculated without taking this into account; this may not always be defined as we saw, but when the raw market value (summing up the prices times the positions) is positive, it is much more intuitive.

In the Index model analytics, you will see both the weights and raw weights, and performance measures calculated with both the (adjusted) weights and raw weights. Others, like the Long-Short Analysis, only report the raw weights because in that context, the long and short positions are separated out and the raw weights are more natural.

Constructing a diversified portfolio

Let $E(r_i)$ denote the expected return from stock i. This is usually measured annually, so $E(r_i)$=10% means you expect the stock to return 10% over the year. There are many ways to estimate the expected return;

stock analysts use techniques involving historical data, fundamental analysis, and scenario analysis. Values based on the CAPM are provided by the FTS Real Time Client, though you can override them via the "Parameters" menu item. Given the portfolio weights, the expected return of the portfolio is:

$$ER(w) = \sum_i w_i E(r_i)$$

Finally, we need to describe risk. Modern portfolio theory uses the standard deviation of returns (also called *volatility*) as a measure of risk (or equivalently, the variance). To calculate the risk of a portfolio, you also need the covariances between stock returns. In notation, let σ_{ij} denote the covariance between the returns of stocks i and j, so σ_{ii} is the variance of the return of stock i. Then, given the portfolio weights, the variance of the portfolio return is:

$$\sigma^2(w) = \sum_i \sum_j w_i w_j \sigma_{ij}$$

The volatility is the square root of this number.

The FTS Real Time Client calculates all the covariances for you. The portfolio selection problem is to find weights that minimize the variance subject to some constraints. The first is that the sum of the weights equals 1; this simply means that you invest all the money you have allocated to stocks. The second is that the expected return from the portfolio equals your desired return. Beyond that, you can impose more conditions. For example, you may restrict short selling, either completely, which says $w_i \geq 0$. Or you may require that you will not invest more some amount in any one stock; this says $w_i \leq 0.1$. Common constraints for this case, which has 30 stocks, would be that you do not invest more than 15% in any one stock, and if there is no short selling, that you invest at least 2% in every stock.

Project

The objective of this project is to learn to apply modern portfolio theory by constructing portfolios, implementing the recommended portfolio, and tracking its performance over time. This is done by completing the following steps, and includes learning how to use Solver to calculate the portfolio weights.

1. From the RT Client, select "Covariances and Returns" in the Analytics area (at the bottom right); you will see your (raw) portfolio weights and the expected returns and covariances of all the stocks. In the Edit menu of the Analytics area, select "Export to Excel." This will transfer the data into an Excel spreadsheet.
    *   You have two choices with the covariances and returns. If you select "Covariances and Returns," it shows you either the default expected return or the value you entered under Parameters. If you select "Covariances and Returns (CAPM)" it shows you the expected return based on CAPM.
2. Decide on a target expected return. This could be a number you pick, e.g. 7%, or you could be guided by the past return of an index such as the S&P500, or the yield on Treasury bonds.

3. Choose what constraints you want to impose. You may want to consult your investments text (e.g. the chapter on Optimal Risk Portfolios in the text by Bodie, Kane, and Marcus) for suggestions.

    - One constraint you could impose is a limit on the maximum amount in any one stock, for example, that $-0.2 <= w_i <= 0.2$ which says that you will not invest more than 20% in any one stock and that any short position will not be more than 20% of the amount invested.

4. In your spreadsheet, implement the formulas for calculating the portfolio's expected return and variance. You may want to use the SUMPRODUCT function in Excel as well as its matrix multiplication functions.

5. Run Solver, define the objective (which is to minimize the variance) and the constraints (the two basic constraints plus additional constraints you chose), and calculate the portfolio weights. Note that if your constraints are unreasonable, there may not be a solution.

    - Note: you may want to conduct this exercise with 5 or 10 stocks only

6. Implement your trading strategy using the Real Time FTS Client.   This means that you have to take the weights and using current prices, calculate the number of shares of each stock to buy (or sell).

    - **Caution**: if you allow short sales, be careful about the total amount of money you will need to finance the short sale. If you solve for raw weights in the spreadsheet, then you are implicitly assuming that the proceeds of the short sale can be used to buy other stocks. When you implement the trade, you will not get the proceeds; in fact you will have to put up a 50% as well. So the amount you have to buy stocks will be less than you think, and you will run out of money!
    - The **Equity Portfolio Rebalancer** built in to the Windows version of the FTS Real Time Client calculates the costs of rebalancing, the cash required to implement the trades, and can also execute the trades for you.

7. Wait a week. At that point, your actual portfolio weights will have changed because stock prices will have changed. You now have to decide whether you want to "rebalance" your position. This means buying or selling to get back to the weights you had originally calculated. Or you may want to repeat steps 1-6 again since some of the return and covariance estimates may have changed. Note that the decision to rebalance is not that simple, since buying and selling stocks will incur a transaction cost; so if your weights are not too far from where they should be, you may want to wait to rebalance.

    - The **Equity Portfolio Rebalancer** lets you determine the costs of rebalancing.

8. After a few weeks, answer the following questions: What was the risk and return of your portfolio over the time horizon? Did it conform to what was expected? Did you have to rebalance frequently?   You should calculate the realized return, the volatility of your returns, and the Sharpe ratio for your portfolio, and compare these to the S&P 500.

Example with the FTS 30 Stock Case

We show you here a detailed example with the FTS 30 Stock case. We will allow short sales to show you how to adjust your spreadsheet appropriately.

First, select "Covariances and Returns" from the analytics area:

Edit ▾	US Dollar	▾	Stocks: Covariances and Returns	▾	Parameters			UserTip

Name	Weight	Expected Return	3M COMPANY	ALCOA	AMERICAN EXPRESS	AT&T	BANK OF AI
Portfolio Exp Return	0.09054						
Portfolio Variance	0.08354	Portfolio Volatility		0.28903			
Target(Benchmark) Return	0.0821						
Sum of Weights	1.00000		Covariance Matrix				
3M COMPANY	0.0000	0.0802	0.0534	0.1391	0.1431	0.0435	
ALCOA	0.0129	0.1465	0.1391	0.3624	0.3727	0.1134	
AMERICAN EXPRESS	0.0951	0.1496	0.1431	0.3727	0.3833	0.1166	
AT&T	0.0000	0.0726	0.0435	0.1134	0.1166	0.0355	
BANK OF AMERICA	0.0313	0.1639	0.1615	0.4208	0.4327	0.1316	
BOEING	0.0000	0.1057	0.0864	0.2250	0.2314	0.0704	
CATERPILLAR	0.0826	0.1317	0.1200	0.3126	0.3215	0.0978	
CHEVRON	0.0000	0.0741	0.0455	0.1185	0.1219	0.0371	
CISCO SYSTEMS	0.0000	0.1021	0.0818	0.2130	0.2190	0.0666	
COCA-COLA	0.0000	0.0705	0.0409	0.1065	0.1095	0.0333	
E.I DU PONT DE NEMOURS	0.0000	0.1149	0.0982	0.2559	0.2632	0.0800	
EXXONMOBIL	0.0718	0.0593	0.0264	0.0687	0.0707	0.0215	
GENERAL ELECTRIC CO	0.0366	0.1261	0.1127	0.2937	0.3020	0.0919	

From the Edit menu circled above, select "Export to Excel" to get the data in a spreadsheet:

The stocks are in rows 7 to 36, the expected returns are in column C and the covariance matrix is in D7:AG36. The weights shown don't matter; we will be calculating new weights.

The first thing we have to do is replace cells B2, B3 and B5 with formulas.

B5 is the sum of the weights; this has to be 1, so enter the formula: **=SUM(B7:B36)**

B2 is the portfolio expected return;, so enter the formula **=SUMPRODUCT(B7:B36,C7:C36)**

B3 is the portfolio variance; this formula is **=SUMPRODUCT(B7:B36,MMULT(D7:AG36,B7:B36))** , which is the Excel implementation of the double-summation formula for the portfolio variance.

Finally, replace D3 with the formula **=SQRT(B3)** so the portfolio volatility is calculated correctly.

What we have done so far is replace the values copied from the FTS Real Time Client into formulas where the values are calculated as a function of the portfolio weights.

When we use Solver to find the portfolio weights, we will be asking it to find the lowest variance portfolio that matches our target return. The target return is in cell B4; initially, it is simply the expected benchmark return, which in the FTS 30 Stock Case is the S&P500 index. You can change that to a higher number if you want to try and beat the benchmark; we changed it to 0.09 for this example.

Now, we have to think about constraints on the problem. The first constraint is that the sum of the weights is 1: **B5 = 1**. The second constraint is that the portfolio expected return equals our target return: **B2 = B4**

We could also decide to disallow short sales; that would be the constraint **B7:B36 >= 0** but we will not impose that. Other constraints you could impose could be limits on allocations, e.g. no more than 10% in any one stock, or no more than 20% in any sector, and so on.

Enter these into Solver and click `Solve'
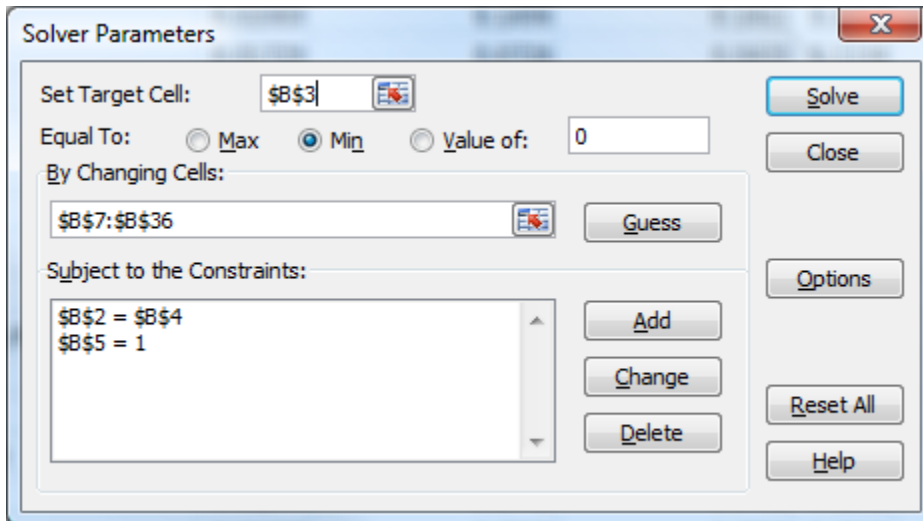
A 20% maximum and minimum on any stock would require adding the constraints $B$7:$B$36 <= 0.2 and $B$7:$B$36 >= -0.2.

For the base case, we obtained a solution:
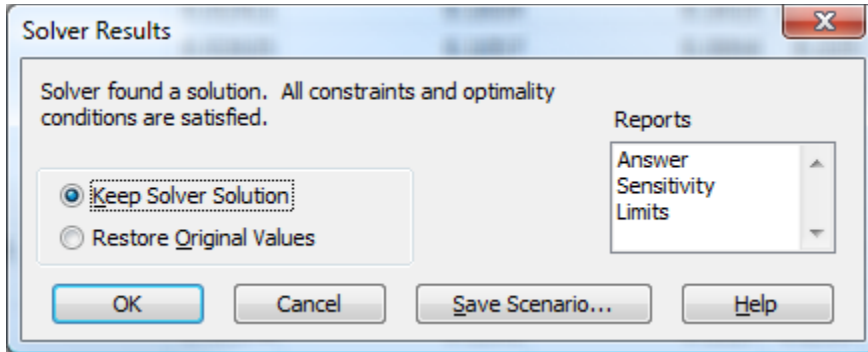
	A	B	C	D	E	F	G	H
1	As of 9/9/2010							
2	Portfolio Exp Return	0.09						
3	Portfolio Variance	0.08392	Portfolio Volatility	0.289694162				
4	Target(Benchmark) Return	0.09						
5	Sum of Weights	1		Covariance Matrix				
6	Name	Weight	Expected Return	3M COMPANY	ALCOA	AMERICAN EXPRESS	AT&T	BANK OF AMERICA
7	3M COMPANY	0.33038	0.0802	0.0572	0.1391	0.1431	0.0435	0.161
8	ALCOA	0.02094	0.1465	0.1391	0.7248	0.3727	0.1134	0.420
9	AMERICAN EXPRESS	0.02069	0.1496	0.1431	0.3727	0.7666	0.1166	0.432
10	AT&T	0.01729	0.0726	0.0435	0.1134	0.1166	0.0709	0.131
11	BANK OF AMERICA	0.01911	0.1639	0.1615	0.4208	0.4327	0.1316	0.977
12	BOEING	0.02635	0.1057	0.0864	0.225	0.2314	0.0704	0.261
13	CATERPILLAR	0.02281	0.1317	0.12	0.3126	0.3215	0.0978	0.362
14	CHEVRON	0.01848	0.0741	0.0455	0.1185	0.1219	0.0371	0.137
15	CISCO SYSTEMS	0.04517	0.1021	0.0818	0.213	0.219	0.0666	0.247
16	COCA-COLA	0.00949	0.0705	0.0409	0.1065	0.1095	0.0333	0.123
17	E.I DU PONT DE NEMOURS	0.02545	0.1149	0.0982	0.2559	0.2632	0.08	0.297
18	EXXONMOBIL	-0.0258	0.0593	0.0264	0.0687	0.0707	0.0215	0.079
19	GENERAL ELECTRIC CO	0.02374	0.1261	0.1127	0.2937	0.302	0.0919	0.34
20	HEWLETT-PACKARD CO	0.02606	0.0899	0.0659	0.1717	0.1766	0.0537	0.199
21	HOME DEPOT INC	0.01874	0.0746	0.0462	0.1202	0.1236	0.0376	0.139
22	INTEL	0.02659	0.0965	0.0745	0.1941	0.1996	0.0607	0.225
23	INTERNATIONAL BUSINESS MACHINE	0.01797	0.0736	0.0448	0.1168	0.1201	0.0365	0.135
24	JOHNSON & JOHNSON	0.25862	0.069	0.0389	0.1013	0.1042	0.0317	0.117
25	J.P. MORGAN CHASE & CO.	0.02773	0.0981	0.0765	0.1992	0.2049	0.0623	0.231
26	KRAFT FOODS	0.02129	0.0685	0.0382	0.0996	0.1024	0.0312	0.115
27	MCDONALDS CORP	-0.0089	0.0664	0.0356	0.0927	0.0954	0.029	0.107
28	MERCK	0.01803	0.0746	0.0462	0.1202	0.1236	0.0376	0.139
29	MICROSOFT	0.02634	0.0919	0.0686	0.1786	0.1837	0.0559	0.207
30	PFIZER	0.01321	0.0731	0.0442	0.1151	0.1183	0.036	0.133
31	PROCTER & GAMBLE	-0.0145	0.0654	0.0343	0.0893	0.0918	0.0279	0.103
32	THE TRAVELERS COMPANIES	0.00964	0.0675	0.0369	0.0962	0.0989	0.0301	0.111

US Dollar_Stocks_ Covariances | Sheet1 | Sheet2 | Sheet3

Notes:

- One technical problem you may face is that the covariance matrix is close to being singular. In this case, Solver may not be able to find a solution without One way you will know that there is a problem is if Solver tells you that the "cell values fail to converge." This is more likely to happen with a large number of stocks. In that case, you will have to modify the covariance matrix in the spreadsheet to make it non-singular.
- The FTS Real Time Client overrides the inputs of a stock's volatility if it is inconsistent. Technically, it increases the volatility internally to make sure that all correlations across stock returns are between -1 and 1. The covariance matrix that is exported to Excel contains this adjustment. If you want to use your own numbers, you can modify them directly in the spreadsheet.
- If you want to use fewer stocks, set the weights of the other stocks to zero, and in Solver, just below it says "By Changing Cells," enter the cells with the weights you want to use separated by commas, e.g., $B$7,$B$9,$B$15
  - Solver is more likely to find a solution with fewer stocks

Now that we have the weights, we can calculate what we have to buy or sell. Suppose that you started off with $1m in cash. Theoretically, if you are supposed to hold w_i of stock i and the price of stock i is P_i, you would buy n_i shares where n_i solves: $w_i = P_i n_i / \$1m$, so $n_i = \$1m * w_i / P_i$. But will not work in practice, for two reasons. First, you will have to pay a transactions cost for each trade; usually, this is fixed at $9.95 a trade; there are also limits on how many stocks you can buy or sell at one time, so you may to make multiple transactions. Second, if you short sell, you will have to put up money and will not be able to use the proceeds of the short sale to buy other stocks.

> **Note:** The **Equity Portfolio Rebalancer** built in to the Windows version of the FTS Real Time Client lets you determine the costs of rebalancing and also can execute the trades for you.

For example, in our solution, the recommended raw weight for Exxon Mobil is -0.0258. At the time we did this, Exxon Mobil was trading at 61.33. So if we had $1m to start, then we want to short 421 shares of Exxon. But this will use up $38,787 of cash.

So you have to first calculate how much of your money would be used up by the short sales and keep some aside for transactions costs. So even though you started with $1m, if the short positions are small, you should think about maybe investing about $900000. With large short positions, you will have to be more careful.

We conclude with some comments:

- You will have to rebalance your portfolio periodically as prices change.
- You may decide to run Solver periodically and recalculate the weights; again, this can be costly so if you plan to do this, make sure you have enough resources available.
- To calculate the results required in Step 5 above, you will need to download your trading history from the Reports menu at the top:
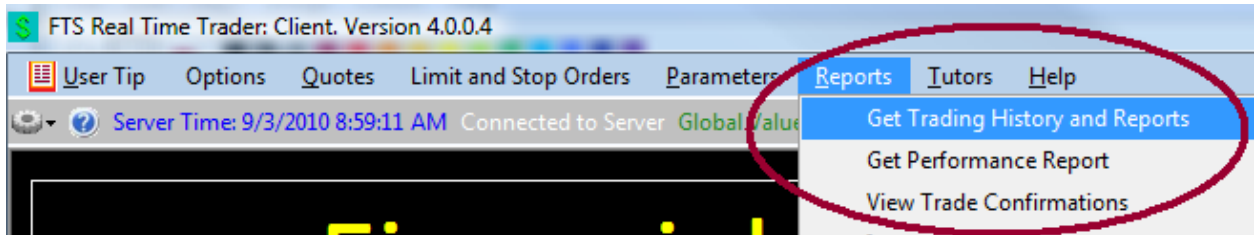
- From the history window, generate the report "End of Day Market Values" and copy the selected information as shown and paste it into Excel:

Date	Value	Return	Benchmark	Benchmark Return	Alpha
8/21/2010	1000000		1071.69		
8/22/2010	1,000,004.00	0.00000	1,071.69	0.00000	0.00000
8/23/2010	1,000,008.00	0.00000	1,067.36	-0.00404	0.00404
8/24/2010	1,000,012.00	0.00000	1,051.87	-0.01451	0.01452
8/25/2010	1,000,017.00	0.00000	1,055.33	0.00329	-0.00329
8/26/2010	999,705.30	-0.00031	1,047.22	-0.00768	0.00737
8/27/2010	1,000,806.00	0.00110	1,064.59	0.01659	-0.01549
8/28/2010	1,000,810.00	0.00000	1,064.59	0.00000	0.00000
8/29/2010	1,000,813.00	0.00000	1,064.59	0.00000	0.00000
8/30/2010	999,839.20	-0.00097	1,048.92	-0.01472	0.01375
8/31/2010	999,783.30	-0.00006	1,049.33	0.00039	-0.00045
9/1/2010	1,002,195.00	0.00241	1,080.29	0.02950	-0.02709
9/2/2010	1,002,097.00	-0.00010	1,090.10	0.00908	-0.00918
9/3/2010	1,003,633.00	0.00153	1,104.51	0.01322	-0.01169
9/4/2010	1,003,636.00	0.00000	1,104.51	0.00000	0.00000
9/5/2010	1,003,639.00	0.00000	1,104.51	0.00000	0.00000

Now, you can calculate what is needed; there is also a performance report that you can generate automatically.