

## FTS DOW School Case (DJIA Stocks) Project: Portfolio Diversification

Note: this project requires use of Excel's Solver

**Question:** How do you create a diversified stock portfolio?

Advice given by most financial advisors is to not put all your eggs in one basket, i.e. to diversify, meaning that you hold several different stocks. The reasoning behind this is simple; if you hold one stock or very few stocks, you can either do very well and very poorly. By diversifying, you can reduce the chances of extreme outcomes, and thus reduce the risk of your portfolio; of course, you typically also affect the return you expect to get. How much you diversify typically depends on your risk aversion; the more you dislike risk, the more you would like to diversify.

Modern portfolio theory provides a technique for both measuring risk and return and determining the best way to diversify. In this project, you will learn to use Excel's Solver to create an "optimal" diversified portfolio. The details of modern portfolio theory are described in most investments textbooks, and will not be given here; instead, we will focus on the implementation, though we will need the following terminology.

### Background

Let  $w_i$  denote the proportion of your money invested in stock  $i$ . So if your total investment is \$1m, you hold 1000 shares of stock  $i$ , and the price of stock  $i$  is 25, then  $w_i = 0.025$ , i.e. you have invested 2.5% of your money in stock  $i$ .  $w_i$  is also called a *portfolio weight*. Since there is a direct relationship between the number of shares and the weight, once you have determined the weight, you can easily calculate the number of shares you must hold (given the price and the total investment).

Let  $E(r_i)$  denote the expected return from stock  $i$ . This is usually measured annually, so  $E(r_i)=10\%$  means you expect the stock to return 10% over the year. There are many ways to estimate the expected return; stock analysts use techniques involving historical data, fundamental analysis, and scenario analysis. Values based on the CAPM are provided by the FTS Real Time Client, though you can override them via the "Parameters" menu item. Given the portfolio weights, the expected return of the portfolio is:

$$ER(w) = \sum_i w_i E(r_i)$$

Finally, we need to describe risk. Modern portfolio theory uses the variance of returns as a measure of risk (or equivalently, the standard deviation, which is also referred to as volatility). To calculate the risk of a portfolio, you also need the covariances between stock returns. In notation, let  $\sigma_{ij}$  denote the covariance between the returns of stocks  $i$  and  $j$ , so  $\sigma_{ii}$  is the variance of the return of stock  $i$ . Then, given the portfolio weights, the variance of the portfolio return is:

$$\sigma^2(w) = \sum_i \sum_j w_i w_j \sigma_{ij}$$

The FTS Real Time Client calculates all the covariances for you. The portfolio selection problem is to find weights that minimize the variance subject to some constraints. The first is that the sum of the weights equals 1; this simply means that you invest all the money you have allocated to stocks. The second is that the expected return from the portfolio equals your desired return. Beyond that, you can impose more conditions. For example, you may restrict short selling, either completely, which says  $w_i \geq 0$ . Or you may require that you will not invest more some amount in any one stock; this says  $w_i \leq 0.1$ . Common constraints for this case, which has 30 stocks, would be that you do not invest more than 15% in any one stock, and if there is no short selling, that you invest at least 2% in every stock.

## Project

The objective of this project is to learn to apply modern portfolio theory by constructing portfolios, implementing the recommended portfolio, and tracking its performance over time. This is done by completing the following steps, and includes learning how to use Solver to calculate the portfolio weights.

1. From the RT Client, select “Covariances and Returns” in the Analytics area (at the bottom right); you will see your portfolio weights and the expected returns and covariances of all the stocks. In the Edit menu of the Analytics area, select “Export to Excel.” This will transfer the data into an Excel spreadsheet.
2. Decide on a target expected return. This could be a number you pick, e.g. 7%, or you could be guided by the past return of an index such as the S&P500, or the yield on Treasury bonds.
3. Choose what constraints you want to impose. You may want to consult your investments text (e.g. the chapter on Optimal Risk Portfolios in the text by Bodie, Kane, and Marcus) for suggestions.
4. In your spreadsheet, implement the formulas for calculating the portfolio’s expected return and variance. You may want to use the SUMPRODUCT function in Excel as well as its matrix multiplication functions.
5. Run Solver, define the objective (which is to minimize the variance) and the constraints (the two basic constraints plus additional constraints you chose), and calculate the portfolio weights. Note that if your constraints are unreasonable, there may not be a solution.
6. Implement your trading strategy using the Real Time FTS Client. This means that you have to take the weights and using current prices, calculate the number of shares of each stock to buy (or sell).
7. Wait a week. At that point, your actual portfolio weights will have changed because stock prices will have changed. You now have to decide whether you want to “rebalance” your position. This means buying or selling to get back to the weights you had originally calculated. Or you may want to repeat steps 1-6 again since some of the return and covariance estimates may have changed. Note that the decision to rebalance is not that simple, since buying and selling stocks



will incur a transaction cost; so if your weights are not too far from where they should be, you may want to wait to rebalance.

8. After a few weeks, answer the following questions: What was the risk and return of your portfolio over the time horizon? Did it conform to what was expected? Did you have to rebalance frequently? You should calculate the realized return, the volatility of your returns, and the Sharpe ratio for your portfolio, and compare these to the S&P 500.